Math L.O.7

Qena Student Club

Concepts

- **1.** Quadratic Function
- 2. First and second differences
- 3. Completing the square
- 4. Complex numbers
- 5. Parabola
- 6. Focus
- 7. Argand diagram
- 8. related roots

Quadratic function

- A quadratic function is a polynomial function of
- degree 2 . So a
- quadratic function is a function of f x = ax^2 + bx + c , **a**≠0
- The solution of a quadratic equation
- Ax^2 + bx + c is given by
- $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- the expression b^2 – 4ac is called the discriminant

- ✓ $D>0$ **then the roots are real and distinct**
- ✓ $D=0$ **then the roots are real and equal**
- ✓ $D<0$ **then the roots are imaginary and distinct**
- ✓ $D>0$
- ✓ $D>0$ **and a perfect square, then the roots are rational**
- ✓ $a=1$, **and not a perfect square, the roots are irrational**
- **b, c ∈Q, and D is a perfect square, then both the roots are integers**
- A+b+c=0, then 1 is one root and the other root will be (ca)
- A+b+c=0, −1is one root and the other root is − ca
- the equation (ax^2 +bx + c)has real roots α
- and β, we write ax^2 +bx + c =ax – a (x-B)
- is a polynomial function of second degree that has almost 2 solution
- solution=zeroes=roots=X-intercept
- they are mean the values of x to make f(x)=0
- When a<0 the function will be downward
- When a>0 the function will be upward

forms of quadratic functions

standard

$y=aX^2+bX+c$

in standard form vertex point(x,y), $x=-b/2a$

$y=f(-b/2a)$

factored

$f(x)=a(x-p)(x-q)$

Solutions of function when function equals 0

Vertex =$(p+q/2 , f(p+q/2))$

vertex

$f(x)=a(x-h)^2+k$

Range = $[k, \infty]$

Range=$[-\infty,k]$

vertex point=(h,k)

Axis of semmetry

x=h

First and second differences

If first differences are equals the function will be liner but if the second which are equals the function will be quadratic function

x	y	First Differences	Second Differences
0	2		
1	3	1	
2	6	3	2
3	11	5	2
4	18	7	2

Complex number

- When you try to solve a quadratic function, you get three
- possibilities, one of them when $D<0$. In this one the
- solutions are imaginary. You must know complex numbers
- to solve these equations.

Two complex numbers are equal if and only if the two real parts are equal and the two imaginary parts are equal.

if: $a+bi=c+di$ then: $a=c$ and $b=d$ and vice versa.

Example:

Find in the simplest form the result of each of the following:

A $(7 - 4i) + (2 + i)$

Solution

The expression $(7 - 4i) + (2 + i)$

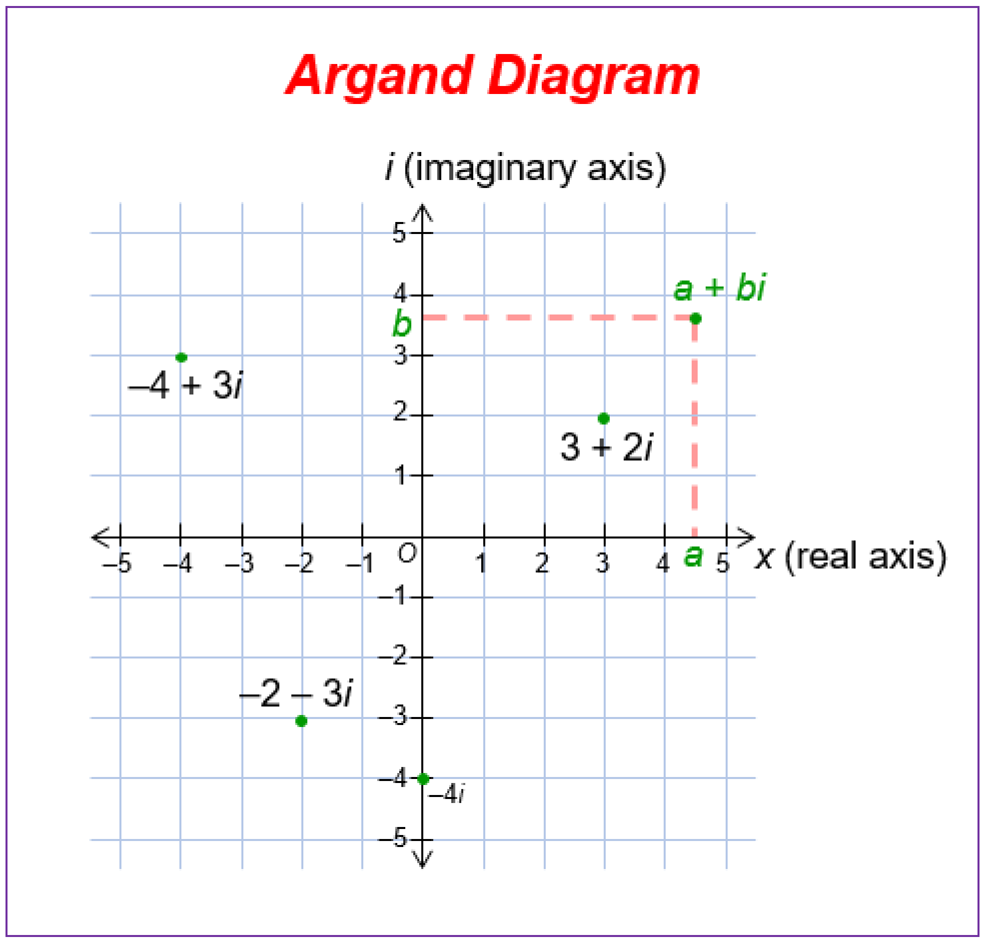
$= (7 + 2) + (-4 + 1)\,i$ Commutative and associative properties

$= 9 - 3\,i$ Simplify

Argand diagram

What is the point on an Argand diagram?

Any complex number z can be represented by a point on an Argand diagram. We can join this point to the origin with a line segment. The length of the line segment is called the modulus of the complex number and is denoted |z|



- **Example 1**

Plot the following complex numbers on an Argand diagram.

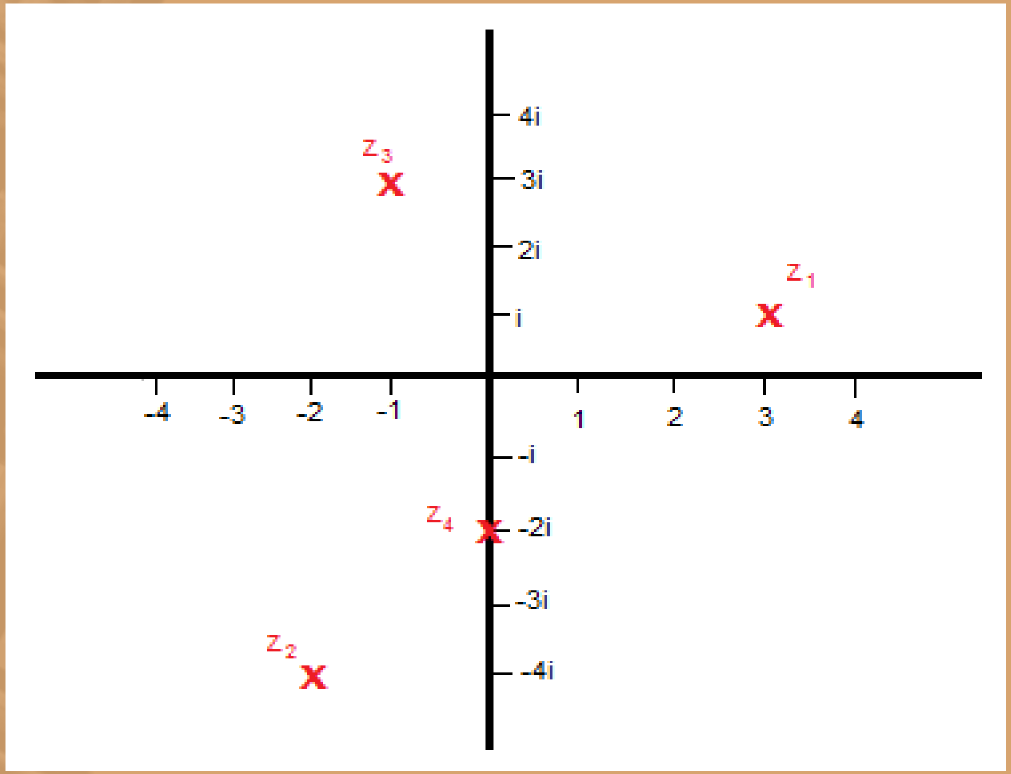
z1=3+I

z3=−1+3i

z4=−2i

z2=−2−4i

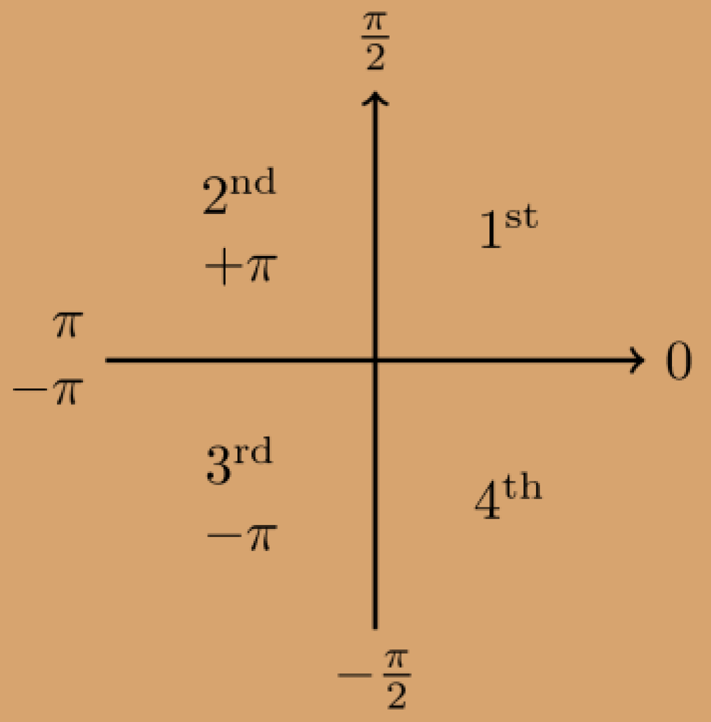
- **Solution**



- The modulus of a complex number $z=a+bi$ is

$$|z|=\sqrt{a2+b}$$

When calculating the argument of a complex number, there is a choice to be made between taking values in the range $[-\pi,\pi]$ or the range $[0,\pi]$ Both are equivalent and equally valid. On this page we will use the convention $-\pi<\theta<\pi$.



The 'naive' way of calculating the angle to a point (a,b) is to use arctan(b/a)arctan but, since arctan only takes values in the range $[-\pi/2,\pi/2]$this will give the wrong result for coordinates with negative x-component.

You can fix this by adding or subtracting π, depending on which quadrant of the Argand diagram the point lies in.

Example

Find the modulus and argument of the complex number z=3+2i

$|z|=\sqrt{32+22}$

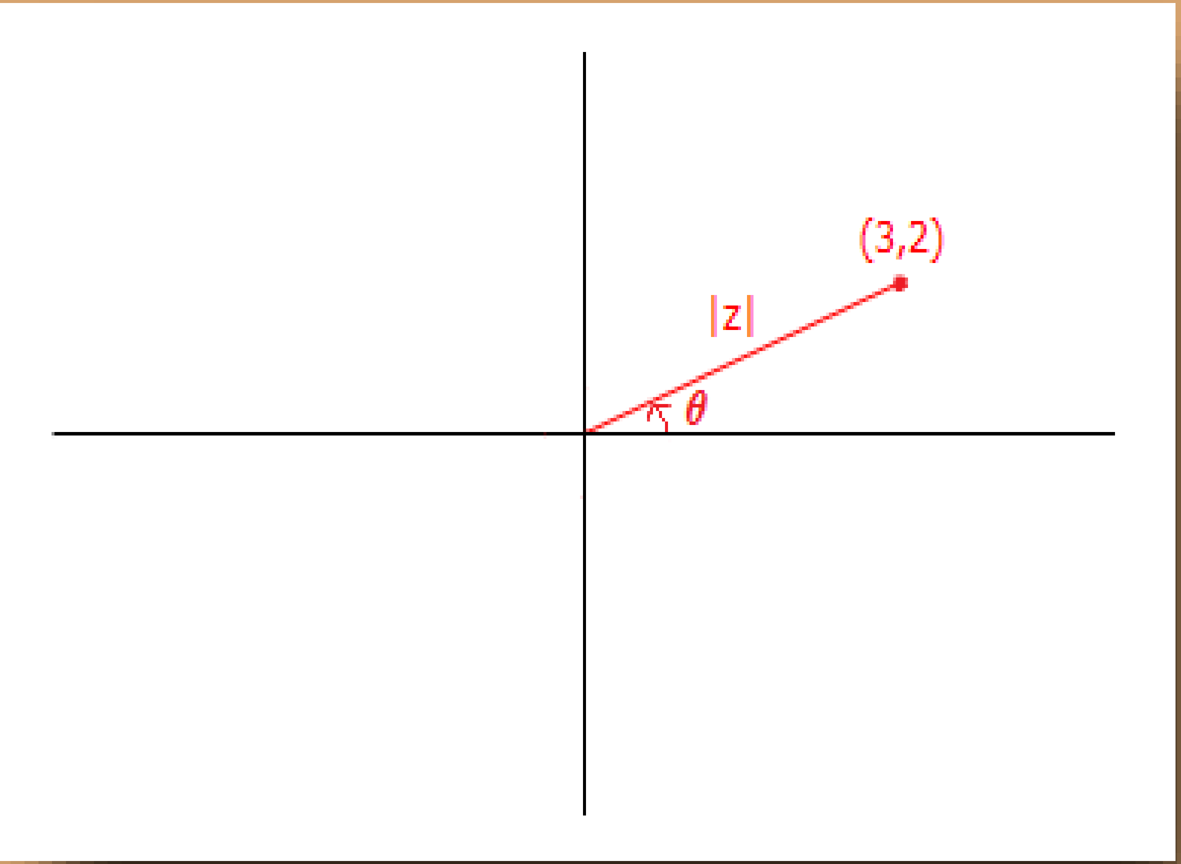
$=\sqrt{9+4}$

$=\sqrt{13}$

As the complex number lies in the first quadrant of the Argand diagram, we can use arctan23arctan23 without modification to find the argument.

$\arg z=\arctan(23)$

$=0.59$ radians (to 2 d.p.)



Completing the square

How do you solve quadratics by completing the square?.

Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Perfect Square Trinomials

Examples

- X2 + 6x + 9
- X2 - 10x + 25
- X2 + 12x + 36

Creating a Perfect Square Trinomial : _

- In the following perfect square trinomial, the constant term is missing.

X2 + 14x + ____

- Find the constant term by squaring half the coefficient of the linear term.

(b/2)2

- (14/2)2

X2 + 14x + 49

Create perfect square trinomials:

- $x^2 + 20x +$ ____
- $x^2 - 4x +$ ____
- $x^2 + 5x +$ ____

Ans:

1-100

2-4

3-25/4

Solving Quadratic Equations by Completing the Square:

Solve the following equation by completing the square:

$$x^2 + 8x - 20 = 0$$

Step 1: Move quadratic term, and linear term to left side of the equation

$$x^2 + 8x = 20$$

Step 2: Find the term that completes the square on the left side of the equation.
Add that term to both sides.

$$x^2 + 8x + \square = 20 + \square$$

$$\frac{1}{2} \bullet (8) = 4 \text{ then square it, } 4^2 = 16$$

$$x^2 + 8x + 16 = 20 + 16$$

Step 3: Factor the perfect square trinomial on the left side of the equation. Simplify the right side of the equation.

$$x^2 + 8x + 16 = 20 + 16$$

$$(x - 4)(x - 4) = 36$$

$$(x - 4)^2 = 36$$

Step 4: Take the square root of each side

$$\sqrt{(x + 4)^2} = \sqrt{36}$$

$$(x + 4) = \pm 6$$

Step 5: Set up the two possibilities and solve

$$x = -4 \pm 6$$

$$x = -4 - 6 \text{ and } x = -4 + 6$$

$$x = -10 \quad \text{and} \quad x=2$$

Completing the Square-Example #2

Solve the equation $x^2 + 8x + 5 = 0$ by completing the square

- **Solution**

First, rewrite the equation in the form $x^2 + bx = c$.

$$x^2 + 8x = -5$$

Add the appropriate constant to complete the square, then simplify.

$$x^2 + 8x + 16 = -5 + 16$$

$$(x + 4)^2 = 11$$

Now solve using the square root method.

Try the following examples. Do your work on your paper and then check your answers.

1. $x^2 + 2x - 63 = 0$

2. $x^2 + 8x - 84 = 0$

3. $x^2 - 5x - 24 = 0$

4. $x^2 + 7x + 13 = 0$

5. $3x^2 + 5x + 6 = 0$

Ans:

1. $(-9, 7)$

2. $(6, -14)$

3. $(-3, 8)$

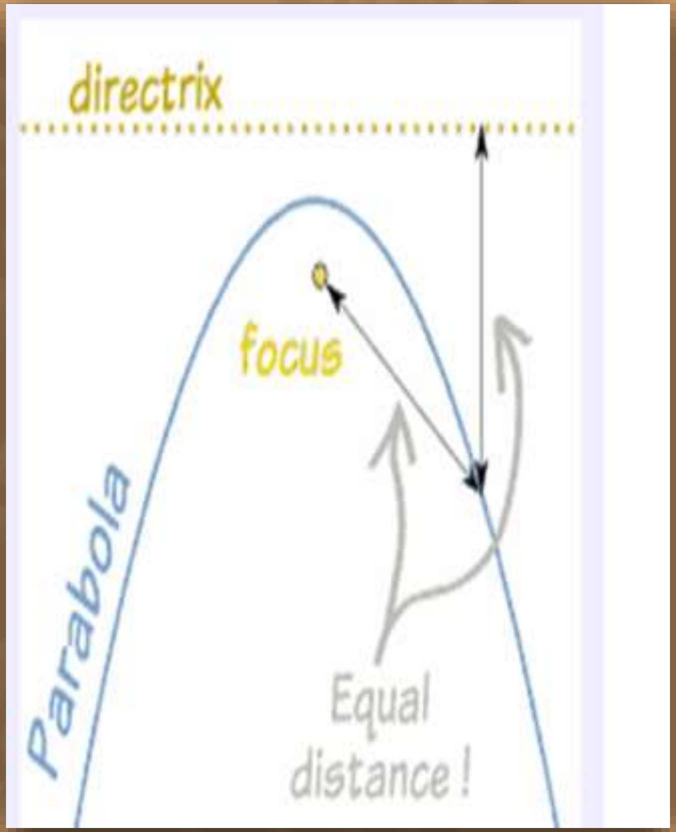
4. $\left(\dfrac{-7 \pm i\sqrt{3}}{2}\right)$

5. $\left(\dfrac{-5 \pm i\sqrt{47}}{6}\right)$

The Parabola

The parabola is the locus of all points in a plane that are the same distance from a line in the plane, the directrix, as from a fixed point in the plane, the focus.

Point Focus = Point Directrix
PF = PD



The parabola has one axis of symmetry, which intersects the parabola at its vertex.

The distance from the directrix to the vertex is also | p |.

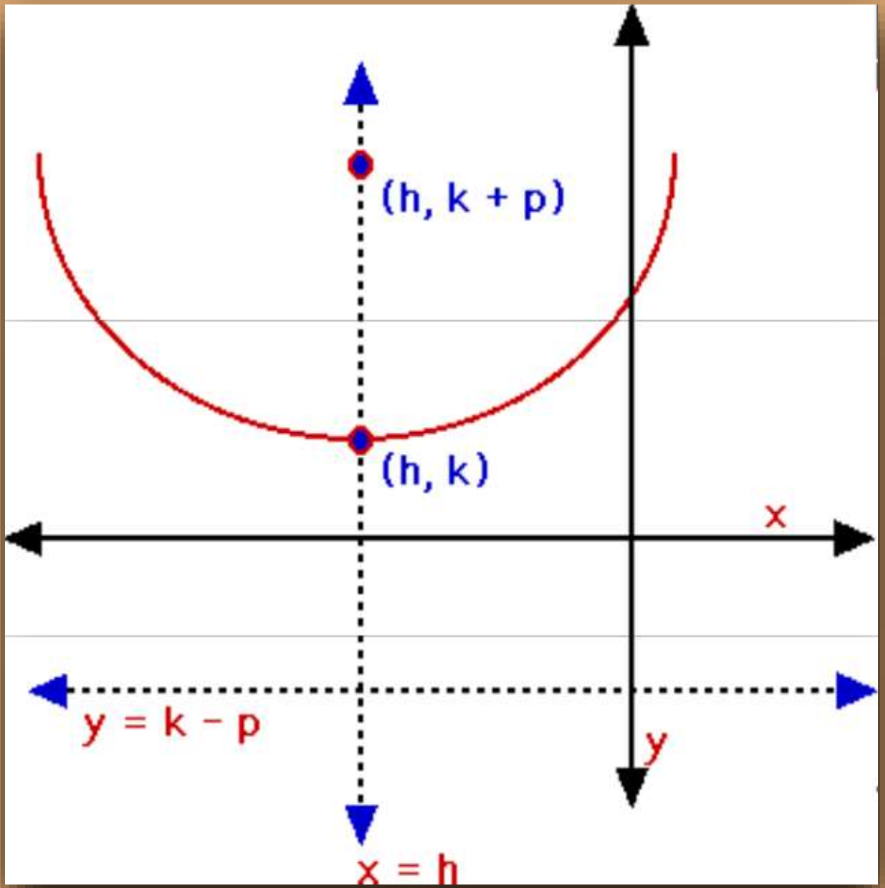
The distance from the vertex to the focus is | p |.

The Standard Form of the Equation with Vertex (*h*, *k*)

- For a parabola with the axis of symmetry parallel to the *y*-axis and vertex at (*h*, *k*), the standard form is …

$$(x - h)^2 = 4p(y - k)$$

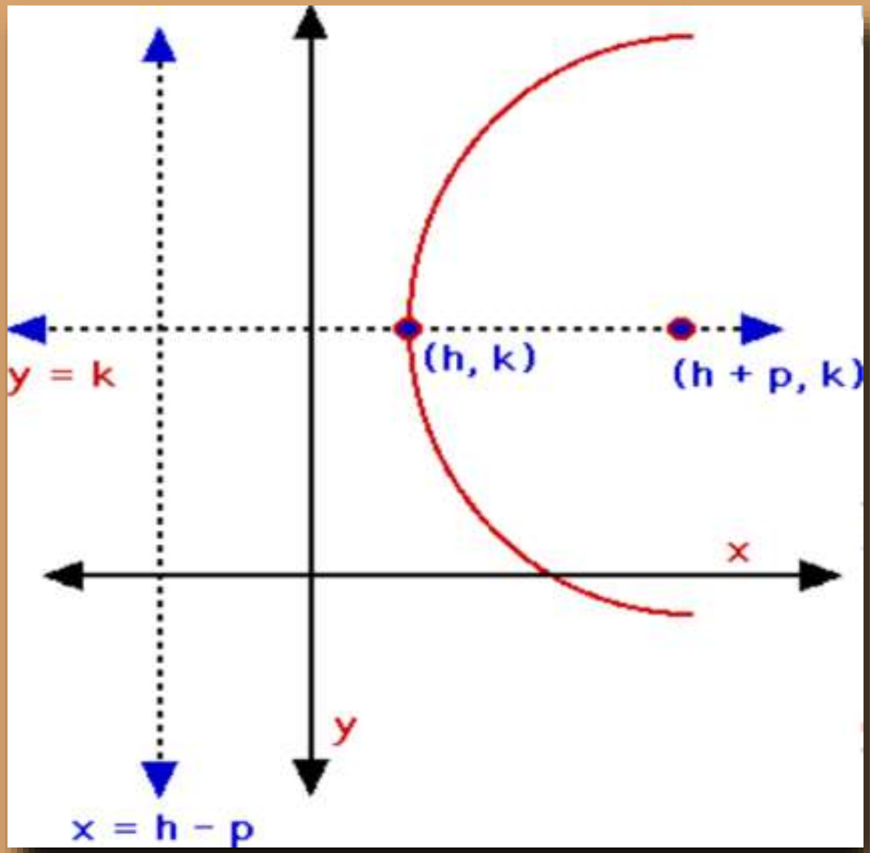
- The equation of the axis of symmetry is $x = h$.
- The coordinates of the focus are $(h, k + p)$.
- The equation of the directrix is $y = k - p$.
- When p is positive, the parabola opens upward.
- When p is negative, the parabola opens downward.



- For a parabola with an axis of symmetry parallel to the *x*-axis and a vertex at (*h, k*), the standard form is:

$$(y - k)^2 = 4p(x - h)$$

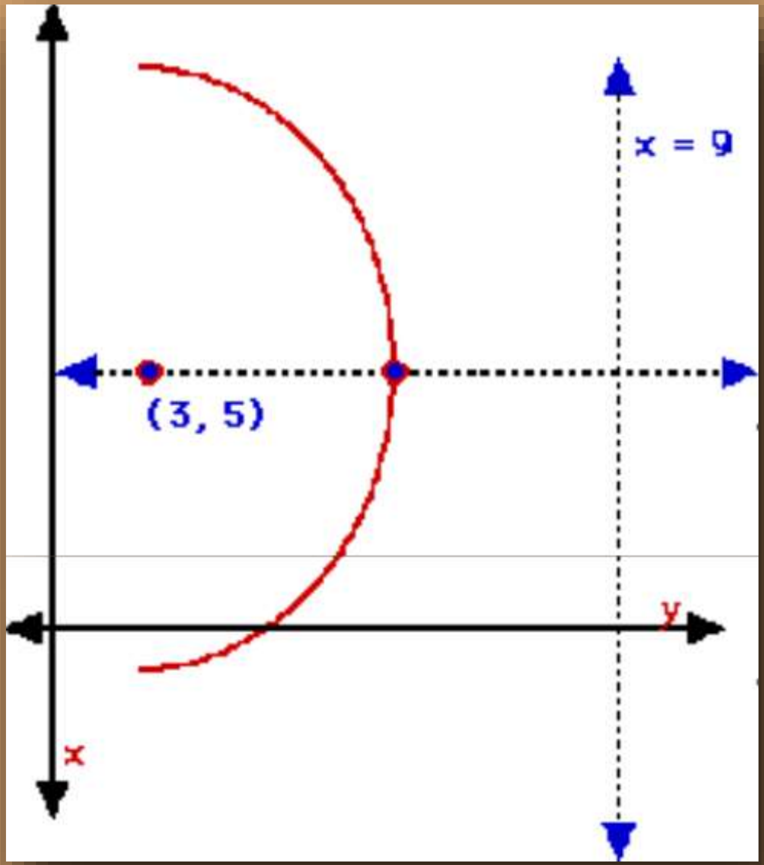
- The coordinates of the focus are (*h* + *p*, *k*).
- The equation of the directrix is *x* = *h* - *p*.
- When *p* is positive, the parabola opens to the right.
- When *p* is negative, the
- parabola opens to the left.



Finding the Equations of Parabolas

Write the equation of the parabola with a focus at (3, 5) and the directrix at $x = 9$, in standard form and general form

The distance from the focus to the directrix is 6 units, therefore, $2p = -6$, $p = -3$. Thus, the vertex is (6, 5).



The axis of symmetry is parallel to the x-axis:

$(y - k)^2 = 4p(x - h)$

$h = 6$ and $k = 5$

$(y - 5)^2 = 4(-3)(x - 6)$

$(y - 5)^2 = -12(x - 6)$

Standard form

Finding the Equations of Parabolas

Find the equation of the parabola that has a minimum at (-2, 6) and passes through the point (2, 8).

The axis of symmetry is parallel to the *y*-axis.

The vertex is (-2, 6), therefore, $h = -2$ and $k = 6$.

$$(x - h)^2 = 4p(y - k)$$

$x = 2$ and $y = 8$

$$(2 - (-2))^2 = 4p(8 - 6)$$

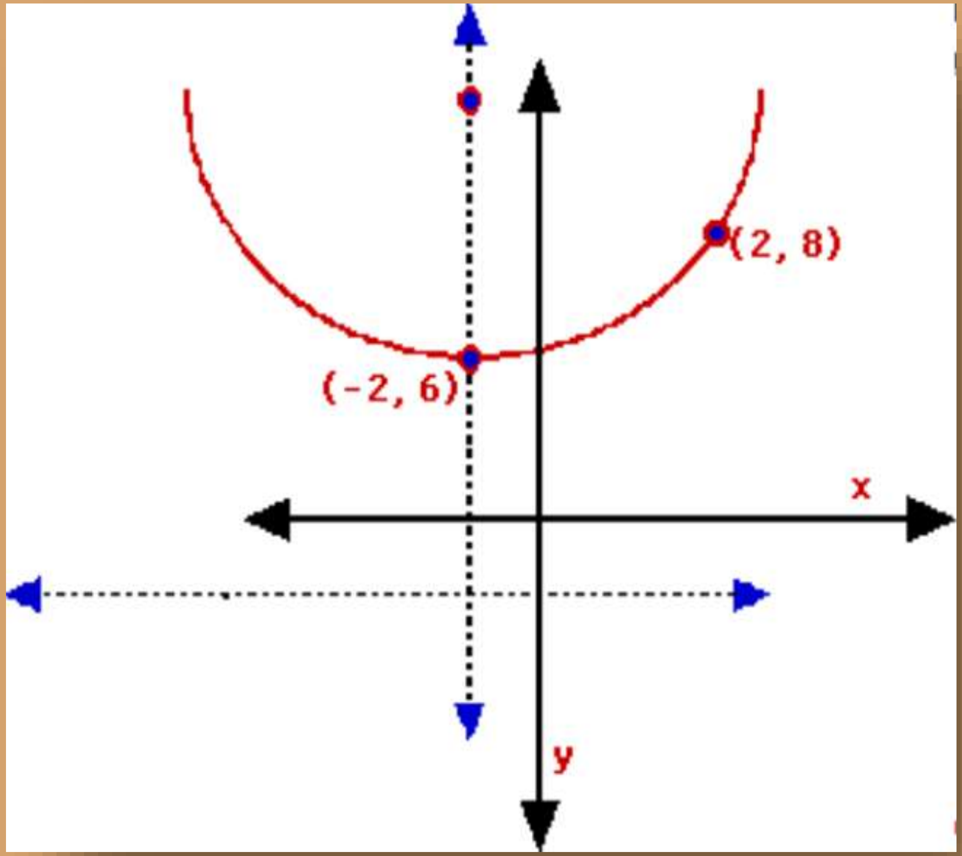
$$16 = \qquad 2 = p$$

$$(x - h)^2 = 4p(y - k$$

$$(x - (-2))^2 = 4(2)(y - 6)$$

$$(x + 2)^2 = 8(y - 6)$$

Standard form



Related Roots

- Note: The discriminant: It tell us how many roots the equation has = b2-4ac
- If b2 -4ac > 0 Two different and real root
- B2 -4ac = 0 Two equal real roots
- B2 - 4ac < 0 Two imaginary roots
- Note: A2+B2= (A+B)2-2AB
- Note: (A-b)2= (A+B)2-4AB